

Due Tuesday, March 26, 2025.

Problem 1 (Thomas Problem §9.1 # 1). Consider the differential equation and the function

$$2y' + 3y = e^{-x} \quad \text{and} \quad f(x) = e^{-x} + 5e^{-(3/2)x}.$$

Show that $y = f(x)$ is a solution to the differential equation.

Problem 2 (Thomas Problem §9.1 # 3). Consider the function and the differential equation

$$f(x) = \frac{1}{x} \int_1^x \frac{e^t}{t} dt \quad \text{and} \quad x^2 y' + xy = e^x.$$

Show that $y = f(x)$ is a solution to the differential equation.

Problem 3 (Thomas Problem §8.1 # 21). Integrate)

$$\int 3^{x+1} dx.$$

Problem 4 (Thomas Problem §8.1 # 77). Integrate

$$\int \frac{6 dy}{\sqrt{y}(1+y)}.$$

Problem 5 (Thomas Problem §8.2 # 27). Integrate

$$\int_0^{\pi/3} x \tan^2 x dx.$$

Problem 6. Compute $\int_1^e \frac{x^2 + 1}{x} dx$.

Problem 7. Compute $\int_0^1 \frac{1}{x^2 + 1} dx$.

Problem 8 (Thomas Problem §8.2 # 33). Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes, the curve $y = e^x$, and the line $x = \ln 2$, about the line $x = \ln 2$.

Problem 9 (Thomas Problem §4.1 # 43). Consider the function

$$f(x) = \frac{x}{x^2 + 1}.$$

Find the extreme values of the function and where they occur.

Problem 10 (Thomas Problem §4.2 # 51). The *geometric mean* of two positive real numbers a and b is \sqrt{ab} . Show that the value of c in the conclusion of the Mean Value Theorem for $f(x) = \frac{1}{x}$ on an interval of positive numbers $[a, b]$ is $c = \sqrt{ab}$.